Name: ______________________ Section: ______________________

The Chain Rule. There are four ways to state it. Two with functions:

$$\frac{d}{dx}\Big[f\big(g(x)\big)\Big] = f'\big(g(x)\big)\cdot g'(x)$$

$$\frac{d}{dx}\Big[f\big(g(x)\big)\Big] = f'\big(g(x)\big)\cdot \frac{d}{dx}\Big[g(x)\Big]$$

and two where u is a "hidden" function of x.

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\Big[f(u)\Big] = f'(u)\cdot\frac{d}{dx}\Big[u\Big]$$

Identify the inside and outside functions, find *both* their derivatives, and apply the chain rule.

1. Let $y=\sqrt{2x+1}$. Find $\frac{dy}{dx}$.

Formula 1:

$$\boxed{\begin{array}{ll} f(u)=\sqrt{u} \Rightarrow & f'(u)=\frac{1}{2}u^{-\frac{1}{2}} \\ g(x)=2x+1 \Rightarrow & g'(x)=2 \end{array}}$$

So $\frac{dy}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}\cdot 2$

$= \frac{1}{\sqrt{2x+1}}$

2. Let $y=(1-x)^{200}$. Find $y'(x)$.

Formula 2:

$$\boxed{\begin{array}{ll} f(u)=u^{200} \Rightarrow & f'(u)=200u^{199} \\ g(x)=1-x & \end{array}}$$

Saved for later!

So $y'(x) = 200(1-x)^{199}\cdot\frac{d}{dx}[1-x]$

$= -200(1-x)^{199}$

3. Let $y=e^{1-x^2}$. Find $\frac{dy}{dx}$.

Formula 3:

$$\boxed{\begin{array}{ll} y(u)=e^u \Rightarrow & \frac{dy}{du}=e^u \\ u(x)=1-x^2 \Rightarrow & \frac{du}{dx}=-2x \end{array}}$$

So $\frac{dy}{dx} = e^u\cdot(-2x)$

$\boxed{\frac{dy}{dx} = -2xe^{(1-x^2)}}$

Identify the outsie and inside functions, find the *outside's* derivative, and apply the chain rule.

1. Let $F(x)=\sin(x^2)$. Find $F'(x)$.

$$\boxed{\begin{array}{l} f(u)=\sin(u) \\ u=x^2 \end{array}}$$

...

2. Let $F(x)=\sin^2(x)=(\sin(x))^2$. Find $F'(x)$.

$$\boxed{\begin{array}{l} f(u)=u^2 \\ u=\sin(x) \end{array}}$$

...

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For each of the following, (a) identify "the outermost thing going on", (b) apply a single derivative rule to rephrase the problem, and (c) repeat till you're done.

1. Let $f(x) = \sqrt{\boxed{e^x + \sin(x) + 5x^3}}$. Find $f'(x)$.

$\sqrt{u}$ is outermost $\Rightarrow$ chain Rule.

$$\boxed{\begin{array}{l} f(u) = \sqrt{u} \quad \Rightarrow f'(u) = \frac{1}{2}u^{-\frac{1}{2}} \\ u = e^x + \sin(x) + 5x^3 \end{array}}$$

$$f'(x) = \frac{1}{2}u^{-\frac{1}{2}} \cdot \frac{d}{dx}\left[e^x + \sin(x) + 5x^3\right]$$

$= \ldots$

2. Let $f(x) = \tan(\boxed{e^x \cdot x^2})$. Find $f'(x)$.

$\tan(u)$ is outermost $\Rightarrow$ chain rule

$$\boxed{\begin{array}{l} f(u) = \tan(u) \Rightarrow f'(u) = \sec^2(u) \\ u = e^x \cdot x^2 \end{array}}$$

$$f'(x) = \sec^2(e^x \cdot x^2) \cdot \frac{d}{dx}\left[e^x \cdot x^2\right]$$ ← requires product rule...

$= \ldots$

3. Let $f(x) = \boxed{x} \cdot \boxed{\sec(e^x)}$. Find $f'(x)$.

product is outermost $\Rightarrow$ product rule first.

$$f'(x) = x \cdot \frac{d}{dx}\left[\sec(e^x)\right] + \sec(e^x) \cdot \frac{d}{dx}[x]$$

↑ requires chain rule

be careful not to miss a u later.

$$\boxed{\begin{array}{l} f(u) = \sec(u) \Rightarrow f'(u) = \sec(u) \cdot \tan(u) \\ u = e^x \end{array}}$$

$= \ldots$

4. Let $f(x) = \sqrt{\boxed{\sin(e^{2x+1})}}$. Find $f'(x)$.

$\sqrt{u}$ is outermost $\Rightarrow$ chain rule

$$\boxed{\begin{array}{l} f(u) = \sqrt{u} \\ u = \sin(e^{2x+1}) \end{array}}$$

you actually need the chain rule 3 times

$\Rightarrow$ use 3 letters: $u = \sin(e^{2x+1})$

$v = e^{2x+1}$

$w = 2x+1.$